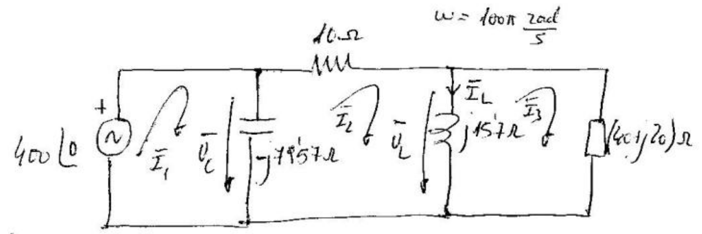
Solución problema 1 de septiembre de 2010.

Conceptos necesarios:

1) Números complejos.
2) RES→ Impedancias, Potencia Activa, Reactiva y Aparente (Compleja)
3) Continuidad
4) Resolución de circuitos de segundo orden. Condiciones de contorno.

Problema 1 (7/9/2010)



$\bar{Z}_C = \frac{1}{jC\omega} = -j\frac{1}{40\cdot10^{-6}\,100\pi} = -j\,79'57\,\Omega$

$\bar{Z}_L = jL\omega = j\,50\cdot10^{-3}\,100\pi = j\,15'7\,\Omega$

(Método de Mallas)

$$\begin{pmatrix} 400\angle 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -j\,79'59 & j\,79'59 & 0 \\ +j\,79'59 & 10-j\,63'89 & -j\,15'7 \\ 0 & -j\,15'7 & 40+35'7j \end{pmatrix} \begin{pmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{pmatrix}$$

$\bar{I}_1 = 18'59\angle{-31'96}$ A

$\bar{I}_2 = 21'67\angle{-40'30}$ A

$\bar{I}_3 = 6'35\angle{4'94}$ A

a) $\bar{S}_E = 400\angle 0 \cdot 18'59\angle 31'96 = \underbrace{7436}_{S_E}\angle 31'96$ VA $= (\underbrace{6308'83}_{P_E} + j\underbrace{3936}_{Q_E})$ VA

b) $\bar{I}_L = \bar{I}_2 - \bar{I}_3 = 21'67\angle{-40'30} - 6'35\angle{4'94} = 17'78\angle{-54'99}$ A

$i_L(t) = 17'78\sqrt{2}\cos(100\pi t - 54'99°)$ A

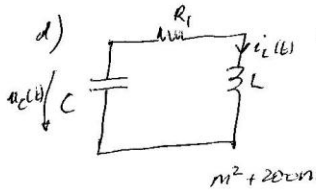
$\bar{U}_L = j\,15'7\cdot(\bar{I}_2 - \bar{I}_3) = j\,15'7\cdot(17'78\angle{-54'99}) = 279'14\angle 35$ V

$u_L(t) = 279'14\sqrt{2}\cos(100\pi t + 35°)$ V

c) $u_C(0^-) = u_C(0^+)$ por continuidad
Como $\bar{u}_C = 400\angle 0 \rightsquigarrow u_C(t) = 400\sqrt{2}\cos(100\pi t)$ $\Big\}$ $u_C(0) = 400\sqrt{2}$ V $= 565'69$ V

$i_L(0^-) = i_L(0^+)$ por continuidad
Como $i_L(t) = 17'78\sqrt{2}\cos(100\pi t - 54'99°)$ $\Big\}$ $i_L(0) = 14'43$ A

d)



Circuito RLC-serie

$LDi_L + \frac{i_L}{CD} + R_1 i_L = 0 \longrightarrow LCD^2 i_L + i_L + R_1 CD i_L = 0 \longrightarrow$

$D^2 i_L + \frac{R_1}{L} D i_L + \frac{1}{LC} i_L = 0 \rightarrow \boxed{D^2 i_L + 200 D i_L + 5\cdot10^5 i_L = 0}$

$m^2 + 200m + 5\cdot10^5 = 0 \longrightarrow m = \frac{-200 \pm \sqrt{200^2 - 4\cdot1\cdot5\cdot10^5}}{2} = \frac{-200 \pm j\,1400}{2} \begin{cases} -100 + j\,700 \\ -100 - j\,700 \end{cases}$

$i_L(t) = e^{-100t}(A\cos 700t + B\,\mathrm{sen}\,700t)$

$i_L(0) = 14'43 = 1\,(A\cdot 1 + B\cdot 0) \longrightarrow \underline{A = 14'43}$

$\underline{i_L(t) = e^{-100t}(14'43\cos 700t + B\,\mathrm{sen}\,700t)}$

$i_L(t) = e^{-100t}(14'43 \cos 700t + B \operatorname{sen} 700t)$

como $R_1 i_L + L D i_L - u_c = 0 \longrightarrow u_c = R_1 i_L + L D i_L$

$$\frac{d i_L}{dt} = -100 e^{-100t}(14'43 \cos 700t + B \operatorname{sen} 700t) + e^{-100t}(-14'43 \cdot 700 \cdot \operatorname{sen} 700t + B \cdot 700 \cdot \cos 700t)$$

$u_c(0^+) = R_1 i_L(0^+) + (L D i_L)_{0^+}$

$565'69 = 10 \cdot 14'43 + 50 \cdot 10^{-3}\left[-100 \cdot (14'43) + 700 B\right]$

$8427'8 = 700B - 1443$

$$\boxed{B = 14'10}$$

luego $\boxed{i_L(t) = e^{-100t}(14'43 \cos 700t + 14'10 \operatorname{sen} 700t)}$