

① $y'' + x^{-1}y = 0, 0 < x < +\infty, \mu > -2.$

req: cambio de variable $t = \frac{x^{\mu/2+1}}{\frac{\mu}{2}+1}$ y transf. $y(x) = Y(t) t^{-\frac{1}{\mu+2}}$

sol: $\{ x^{1/2} J_{\frac{1}{\mu+2}} \left(\frac{2}{\mu+2} x^{\frac{\mu+2}{2}} \right), x^{1/2} Y_{\frac{1}{\mu+2}} \left(\frac{2}{\mu+2} x^{\frac{\mu+2}{2}} \right) \}$

② $xy'' + 2y' + \lambda xy = 0, 0 < x < +\infty, \lambda \in \mathbb{R}.$

req: transf. $Y(u) = \frac{y(x)}{x}$

sol: $\lambda > 0 \Rightarrow y(x) = \frac{A \cos(\sqrt{\lambda}x)}{x} + \frac{B \sin(\sqrt{\lambda}x)}{x}$

$\lambda = 0 \Rightarrow y(x) = A + Bx$

$\lambda < 0 \Rightarrow y(x) = \frac{A}{x} e^{\sqrt{\lambda}x} + \frac{B}{x} e^{-\sqrt{\lambda}x}$

③ $x^2 y'' + (b^2 x^{2b} + \frac{1}{4} - a^2 b^2) y = 0, 0 < x < +\infty, x \in \mathbb{R}^+, b \in \mathbb{N} \setminus \{0\}.$

req: cambio de variable $t = x^b$ y transformación $u(t) = x(t)^{-1/2} y(x(t))$

sol: $\{ x^{1/2} J_{\alpha}(x^b), x^{1/2} Y_{\alpha}(x^b) \}$

④ $(1-x^2)y'' - xy' + u^2 y = 0, x \in (0,1).$

req: cambio de variable $x = \cos t$

sol: $u \neq 0 \Rightarrow y(x) = A \cos(u \arccos x) + B \sin(u \arccos x)$

$u = 0 \Rightarrow y(x) = A + B \arccos x$

⑤ $4y'' + 9xy = 0, x > 0$

req: Hacer primero la transf. $Y(u) = \sqrt{x} y(x)$ y luego el cambio de variable $x^{3/2} = t$

sol: $y(x) = C_1 \sqrt{x} J_{1/3}(x^{3/2}) + C_2 \sqrt{x} Y_{1/3}(x^{3/2})$

⑥ $y'' + e^{-2t} y = 0$ con cambio de variable $s = \frac{2}{e} e^{-\frac{e}{2}t}$

sol: $y(t) = C_1 J_0 \left(\frac{2}{e} e^{-\frac{e}{2}t} \right) + C_2 Y_0 \left(\frac{2}{e} e^{-\frac{e}{2}t} \right)$