① Considerar $A = \begin{pmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

(i) Hallar A^n y $B = \lim_{n\to+\infty} \frac{A^n}{n^2}$

(b) Resolver $\begin{cases} \frac{dx}{dt} = Bx + b(t) \\ x(0) = x_0 \end{cases}$ donde $\begin{cases} b(t) = (0,0,0,\operatorname{sen} t)^T \\ x_0 = (0,0,1,0)^T \end{cases}$

Solución: a) $A^n = \begin{pmatrix} 1 & n & \frac{n^2-n}{2} & \left(n - \frac{n^2-n}{2}\right) \\ 0 & 1 & n & -n \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$B = \begin{pmatrix} 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

b) $X = \begin{pmatrix} \frac{1}{2}t + \frac{1}{2}\operatorname{sen} t \\ 0 \\ 1 \\ 1 - \cos t \end{pmatrix}$

② Resolver $\begin{cases} x_1' = i x_1 + x_2 \\ x_2' = i x_2 + x_3 \\ \dots \\ x_{n-1}' = i x_{n-1} + x_n \\ x_n' = i x_n \end{cases}$ con conds. inics $\begin{cases} x_1(0) = 0 \\ x_2(0) = 0 \\ \dots \\ x_{n-1}(0) = 0 \\ x_n(0) = 1 \end{cases}$

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solución: $X(t) = \begin{pmatrix} \frac{t^{n-1}}{(n-1)!} e^{it} \\ \vdots \\ t e^{it} \\ e^{it} \end{pmatrix}$

③ Resolver $\begin{cases} \frac{dY}{dx} = \begin{pmatrix} 1 & 0 \\ 100\cos x & 1 \end{pmatrix}^{1/100} \cdot Y + \begin{pmatrix} e^x \\ 0 \end{pmatrix}, \quad -\infty < x < +\infty \\ Y(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{cases}$

Solución: $Y(x) = \begin{pmatrix} (x+1)e^x \\ e^x((x+1)\operatorname{sen} x + \cos x - 1) \end{pmatrix}$

(4) Resolver
$$\begin{cases} \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1/x^2 & 1/x \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2/x \end{pmatrix}, \quad 0 < x < +\infty \\ \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases}$$

Solución:
$$\begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}x + x^3/2 \\ -\frac{1}{2} + \frac{3}{2}x^2 \end{pmatrix}$$

(5) solución de
$$e^{X} = \operatorname{sen} \begin{pmatrix} -i\pi/2 & 0 \\ 1 & -i\pi/2 \end{pmatrix}$$

Solución:
$$X = \begin{pmatrix} \frac{e^{\pi/2} - e^{-\pi/2}}{2} - i\pi/2 & 0 \\ \frac{e^{\pi/2} + e^{\pi/2}}{e^{\pi/2} - e^{\pi/2}}\, i & \frac{e^{\pi/2} - e^{-\pi/2}}{2} - \frac{i\pi}{2} \end{pmatrix} =$$
$$= \begin{pmatrix} \operatorname{senh}(\pi/2) - \frac{i\pi}{2} & 0 \\ \frac{i}{\operatorname{tgh}(\pi/2)} & \operatorname{senh}(\pi/2) - i\frac{\pi}{2} \end{pmatrix}$$

(NOTA: la matriz es una caja de Jordan !!)

(6)
$$\begin{cases} y'(x) = \lim_{n\to+\infty} \begin{pmatrix} -10^{-n} & n\,10^{-n} \\ 0 & -10^{-n} \end{pmatrix}^{1/n} \cdot y(x), \quad -\infty < x < +\infty \\ y(0) = \begin{pmatrix} 0 \\ 10 \end{pmatrix} \end{cases}$$

Sugerencia: $z^{1/n} = e^{1/n \log_\pi(z)}$.

Solución:
$$y(x) = \begin{pmatrix} 10x\,e^{x/10} \\ 10e^{x/10} \end{pmatrix}$$

(7) Resolver:
$$\begin{cases} \frac{dy}{dx} = \begin{pmatrix} 0 & -1 \\ -1 & 2i \end{pmatrix} y \\ y(0) = \begin{pmatrix} 1 \\ i \end{pmatrix} \end{cases}$$

Solución:
$$y(x) = e^{ix} \begin{pmatrix} 1 - 2xi \\ i - 2x \end{pmatrix}$$