6-7-2007 (Ind)

P-2 Hallar la sol general del sistema de ec. dif

$$\begin{cases} y_1' = y_2 \\ y_2' = -\dfrac{x^2+x-8}{x^2} y_1 + \dfrac{2x+1}{x} y_2 \end{cases} , \quad x \in ]0,\infty[$$

(Sus. - - - )

$y = y_1$ , $y' = y_2$ , $y'' = y_2' = -\dfrac{x^2+x-8}{x^2} y + \dfrac{2x+1}{x} y'$

$$y'' + \frac{-2x-1}{x} y' + \frac{x^2+x-8}{x^2} y = 0$$

$$y = u\cdot v \; , \; y' = u'v + uv' \; , \; y'' = u''v + 2u'v' + uv''$$

Así

$$u''v + 2u'v' + uv'' + \frac{-2x-1}{x}(u'v+uv') + \frac{x^2+x-8}{x^2} u\cdot v = 0$$

$$u\cdot v'' + \left(2u' + \frac{-2x-1}{x} u\right)v' + \left(u'' + \frac{-2x-1}{x} u' + \frac{x^2+x-8}{x^2} u\right) v = 0$$

Coef de $v'$ igual a 0

$$2u' + \frac{-2x-1}{x} u = 0 \rightarrow \frac{du}{dx} = \frac{2x+1}{2x} u \quad \text{V. Sep}$$

$$\int \frac{du}{u} = \int \frac{2x+1}{2x} dx + C \rightarrow \log u = \int \left(1 + \frac{1}{2x}\right) dx + C$$

$$\log u = x + \frac{1}{2}\log x + C \rightarrow u = k\sqrt{x}\, e^x$$

Así $u' = k\left[\dfrac{1}{2\sqrt{x}} e^x + \sqrt{x} e^x\right]$ , $u'' = k\left[\dfrac{-1}{4\sqrt{x^3}} e^x + \dfrac{1}{\sqrt{x}} e^x + \sqrt{x} e^x\right]$

Así

$$k\sqrt{x}e^x v'' + \left(k\left(\frac{-1}{4\sqrt{x^3}} e^x + \frac{1}{\sqrt{x}} e^x + \sqrt{x}e^x\right) + \frac{-2x-1}{x} k\left[\frac{1}{2\sqrt{x}} e^x + \sqrt{x}e^x\right] + \right.$$

$$\left. + \frac{x^2+x-8}{x^2} k\sqrt{x}e^x\right) v = 0$$

Multiplico por $\dfrac{1}{k\sqrt{x}e^x}$

$$\rightarrow v'' + \left( \frac{-1}{4x^2} + \frac{1}{x} + 1 + \frac{-2x-1}{x}\left(\frac{1}{2x}+1\right) + \frac{x^2-\cancel{8}+8}{x^2} \right) v =$$

$$v'' + \frac{-1 + \cancel{4x} + \cancel{4x^2} - \cancel{4x} - 2 - \cancel{8x^2} - \cancel{4x} + \cancel{4x^2} + \cancel{4x} + 32}{4x^2} \cdot v = 0$$

$$v'' - \frac{35}{4x^2} v = 0$$

$$x^2 v'' - \frac{35}{4} v = 0 \quad \text{Ec de euler}$$

$x \in ]0, \infty[$

Polinomio indicial

$$\varphi(\delta) = -\frac{35}{4} + \delta(\delta-1) = \delta^2 - \delta - \frac{35}{4} = 0$$

$$4\delta^2 - 4\delta - 35 = 0$$

$$\delta = \frac{4 \pm \sqrt{16 + 16 \cdot 35}}{8} = \frac{4 \pm 4 \cdot 6}{8} = \frac{4 \pm 24}{8}$$

$$\nearrow \delta = \frac{28}{8} = \frac{14}{4} = \frac{7}{2}$$

$$\searrow \delta = \frac{-20}{8} = \frac{-10}{4} = \frac{-5}{2}$$

$$\text{SFS} \quad \left\{ x^{\frac{7}{2}}, \; x^{-\frac{5}{2}} \right\}$$

$$v = C_1 x^{\frac{7}{2}} + C_2 x^{-\frac{5}{2}}$$

Asi $y = u \cdot v = k\sqrt{x} e^x \left( C_1 x^{\frac{7}{2}} + C_2 x^{-\frac{5}{2}} \right) \Rightarrow$

$$y = K_1 x^4 e^x + K_2 x^{-2} e^x$$

$$y_1 = K_1 x^4 e^x + K_2 x^{-2} e^x$$

$$y_2 = y_1' = K_1 (4x^3 e^x + x^4 e^x) + K_2 (-2x^{-3} e^x + x^{-2} e^x)$$

$$\rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = K_1 \begin{pmatrix} x^4 e^x \\ 4x^3 e^x + x^4 e^x \end{pmatrix} + K_2 \begin{pmatrix} x^{-2} e^x \\ -2x^{-3} e^x + x^{-2} e^x \end{pmatrix}$$