Utilizando la Transformada de Laplace resolver el ~~sistema~~. problema.

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 1 \\ t \end{pmatrix}, \quad 0 \le t < \infty. \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Sup. $\exists$ sol. $y_1, y_2 \in C^1[0, \infty[$ / $\exists M > 0, \exists K \in \mathbb{R}$

con $|y_i(t)|, |y_i'(t)| \le Me^{Kt}$, $i = 1, 2$.

Def. los originales.

$\varphi_i(t) = y_i(t) H(t)$, $\varphi_i'(t) = y_i'(t) H(t)$, $i = 1, 2$.

$f(t) = H(t)$, $g(t) = t H(t)$.

El sist. queda.

$$\begin{pmatrix} \varphi_1'(t) \\ \varphi_2'(t) \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \varphi_1(t) \\ \varphi_2(t) \end{pmatrix} + \begin{pmatrix} f(t) \\ g(t) \end{pmatrix}$$

$$\begin{cases} \varphi_1'(t) = 3\varphi_1(t) - \varphi_2(t) + f(t) \\ \varphi_2'(t) = \varphi_1(t) + \varphi_2(t) + g(t) \end{cases}$$

Tomo transf. de Laplace.

$$pL\varphi_1(p) - \varphi_1(0+) = 3L\varphi_1(p) - L\varphi_2(p) + Lf(p)$$

$$pL\varphi_2(p) - \varphi_2(0+) = L\varphi_1(p) + L\varphi_2(p) + Lg(p)$$

$\varphi_1(0+) = y_1(0) = 1$, $\varphi_2(0+) = y_2(0) = 0$

$Lf(p) = \frac{1}{p}$, $Lg(p) = \frac{1}{p^2}$

$$\rightarrow \begin{cases} (p-3)L\varphi_1(p) + L\varphi_2(p) = \frac{1}{p} + 1 = \frac{p+1}{p} \\ -L\varphi_1(p) + (p-1)L\varphi_2(p) = \frac{1}{p^2} \end{cases}$$

Cramer

$$\mathcal{L}\varphi_1(p) = \frac{\begin{vmatrix} \frac{p+1}{p} & 1 \\ \frac{1}{p^2} & p-1 \end{vmatrix}}{\begin{vmatrix} p-3 & 1 \\ -1 & p-1 \end{vmatrix}} = \frac{\frac{p^2-1}{p} - \frac{1}{p^2}}{\frac{p^2-4p+3+1}{(p-2)^2}} = \frac{p^3-p-1}{p^2(p-2)^2}$$

$$\mathcal{L}\varphi_2(p) = \frac{\begin{vmatrix} p-3 & \frac{p+1}{p} \\ -1 & \frac{1}{p^2} \end{vmatrix}}{(p-2)^2} = \frac{\frac{p-3}{p^2} + \frac{p+1}{p}}{(p-2)^2} = \frac{p-3+p^2+p}{p^2(p-2)^2} =$$

$$= \frac{p^2+2p-3}{p^2(p-2)^2}$$

$t>0$

φ_1 y φ_2 son de V.A. en un entorno de t., y cont en t.

Cont F. inversa.

$$\varphi_1(t) = \lim_{p\to 0} \frac{d}{dp}\left(\cancel{p^2}\, \frac{p^3-p-1}{\cancel{p^2}(p-2)^2} e^{pt}\right) + \lim_{p\to 2} \frac{d}{dp}\left(\cancel{(p-2)^2}\, \frac{p^3-p-1}{p^2\cancel{(p-2)^2}} e^{pt}\right) =$$

$$= \lim_{p\to 0}\left(\frac{(3p^2-1)(p-2)^2 - 2(p-2)(p^3-p-1)}{(p-2)^4} + \frac{p^3-p-1}{(p-2)^2}\, t\right)e^{pt} +$$

$$+ \lim_{p\to 2}\left(\frac{(3p^2-1)\,p^2 - 2p(p^3-p-1)}{p^4} + \frac{p^3-p-1}{p^2}\, t\right)e^{pt} =$$

$$= -\frac{1}{2} - \frac{1}{4}t + \frac{3}{2}e^{2t} + \frac{5}{4}te^{2t}$$

$$\varphi_2(t) = \lim_{p\to 0} \frac{d}{dp}\left(\cancel{p^2}\, \frac{p^2+2p-3}{\cancel{p^2}(p-2)^2} e^{pt}\right) + \lim_{p\to 2} \frac{d}{dp}\left(\cancel{(p-2)^2}\, \frac{p^2+2p-3}{p^2\cancel{(p-2)^2}} e^{pt}\right)$$

$$= \lim_{p\to 0}\left(\frac{(2p+2)(p-2)^2 - 2(p-2)(p^2+2p-3)}{(p-2)^4} + \frac{p^2+2p-3}{(p-2)^2}\, t\right)e^{pt} +$$

$$+ \lim_{p\to 2}\left(\frac{(2p+2)\,p^2 - 2p(p^2+2p-3)}{p^4} + \frac{p^2+2p-3}{p^2}\, t\right)e^{pt} =$$

$$= -\frac{1}{4} - \frac{3}{4}t + \frac{1}{4}e^{2t} + \frac{5}{4}te^{2t}$$

Así

$$y_1(t) = -\frac{1}{2} - \frac{1}{4}t + \frac{3}{2}e^{2t} + \frac{5}{4}te^{2t} \quad \forall t \geq 0$$

$$y_2(t) = -\frac{1}{4} - \frac{3}{4}t + \frac{1}{4}e^{2t} + \frac{5}{4}te^{2t}$$

(por continuid en $t=0$

Utilizando la transformada de Laplace resolver el sistema:

$$(2x'' - x' + 9x) - (y'' + y' + 3y) = 0 \qquad 0 \le t < \infty$$

$$(2x'' + x' + 7x) - (y'' - y' + 5y) = 0$$

con las cond. iniciales

$$x(0) = x'(0) = 1, \quad y(0) = y'(0) = 0$$

Sup. que existe sol. $x, y \in C^1[0, \infty)$ cumpliendo $\exists M > 0$, $\exists k \in \mathbb{R}$ tal que $|x(t)| = |x'(t)| = |x''(t)| = |y(t)| = |y'(t)| = |y''(t)| \le Me^{kt}$, si $t \ge t_0 > 0$

Defino la originales

$$\varphi_1(t) = x(t)H(t), \quad \varphi_1'(t) = x'(t)H(t), \quad \varphi_1''(t) = x''(t)H(t)$$

$$\varphi_2(t) = y(t)H(t), \quad \varphi_2'(t) = y'(t)H(t), \quad \varphi_2''(t) = y''(t)H(t)$$

Así el sistema queda:

$$2\varphi_1''(t) - \varphi_1'(t) + 9\varphi_1(t) - \varphi_2''(t) - \varphi_2'(t) - 3\varphi_2(t) = 0$$

$$2\varphi_1''(t) + \varphi_1'(t) + 7\varphi_1(t) - \varphi_2''(t) + \varphi_2'(t) - 5\varphi_2(t) = 0$$

Tomo transf. de Laplace,

$$2(p^2 L\varphi_1(p) - (\varphi_1'(0+) + p\varphi_1(0+)) - pL\varphi_1(p) + \varphi_1(0+) + 9L\varphi_1(p)$$

$$- p^2 L\varphi_2(p) + (\varphi_2'(0+) + p\varphi_2(0+)) - pL\varphi_2(p) + \varphi_2(0+) - 3L\varphi$$

$$=$$

$$2(p^2 L\varphi_1(p) - (\varphi_1'(0+) + p\varphi_1(0+)) + pL\varphi_1(p) - \varphi_1(0+) + 7L\varphi_1(p) -$$

$$- p^2 L\varphi_2(p) + (\varphi_2'(0+) + p\varphi_2(0+)) + pL\varphi_2(p) - \varphi_2(0+) - 5L\varphi_2(p) =$$

$\varphi_1(0+) = x(0) = 1$, $\varphi_1'(0+) = x'(0) = 1$, $\varphi_2(0+) = y(0) = 0$, $\varphi_2'(0+) = y$

Así

$$L\varphi_1(p)(2p^2 - p + 9) + L\varphi_2(p)(-p^2 - p - 3) = 2p + 1$$

$$L\varphi_1(p)(2p^2 + p + 7) + L\varphi_2(p)(-p^2 + p - 5) = 2p - 3$$

Resolvemos el sistema por Cramer

$$L\varphi_1(p) = \frac{\begin{vmatrix} 2p+1 & -p^2-p-3 \\ 2p+3 & -p^2+p-5 \end{vmatrix}}{\begin{vmatrix} 2p^2-p+9 & -p^2-p-3 \\ 2p^2+p+7 & -p^2+p-5 \end{vmatrix}} = \frac{6p^2+4}{6p^3-6p^2+24p-24} = \frac{1}{3}\frac{3p^2+2}{(p-1)(p^2}$$

$$L\varphi_2(p) = \frac{\begin{vmatrix} 2p^2-p+9 & 2p+1 \\ 2p^2+p+7 & 2p+3 \end{vmatrix}}{6p^3-6p^2+24p-24} = \frac{20}{6p^3-6p^2+24p-24} = \frac{10}{3}\frac{1}{(p-1)(p^2+4)}$$

$t>0$, φ_1, φ_2 de VA localmente $\supset$ cont. ent.

$$\varphi_1(t) = \lim_{p\to 1} \frac{1}{3}\frac{3p^2+2}{p^2+4}e^{pt} + \lim_{p\to 2i}\frac{1}{3}\frac{3p^2+2}{(p-1)(p+2i)}e^{pt} +$$

$$+\lim_{p\to -2i}\frac{1}{3}\frac{3p^2+2}{(p-1)(p-2i)}e^{pt} =$$

$$= \frac{1}{3}e^t + \frac{1}{3}\frac{-12+2}{(2i-1)\,4i}e^{2it} + \frac{1}{3}\frac{-12+2}{(-2i-1)(-4i)}e^{-2it} =$$

$$= \frac{1}{3}\left(e^t + \frac{+10}{+8+4i}e^{2it} - \frac{10}{-8+4i}e^{-2it}\right) =$$

$$= \frac{1}{3}\left(e^t + \frac{80-40i}{80}(\cos 2t + i\,\mathrm{sen}\,2t) + \frac{80+40i}{80}(\cos 2t - i\,\mathrm{sen}\,2t\right.$$

$$= \frac{1}{3}\left(e^t + 2\cos 2t + \mathrm{sen}\,2t\right)$$

$$\varphi_2(t) = \lim_{p\to 1}\frac{10}{3}\frac{1}{p^2+4}e^{pt} + \lim_{p\to 2i}\frac{10}{3}\frac{1}{(p-1)(p^2+2i)}e^{pt} + \lim_{p\to -2i}\frac{10}{3}\frac{1}{(p-1)(p-2i)}e^{pt}$$

$$= \frac{1}{3}\left(2e^t + \underbrace{\frac{10}{(2i-1)\,4i}}_{=\frac{10}{-4i-8}}e^{2it} + \underbrace{\frac{10}{(-2i-1)(-4i)}}_{=4i-8}e^{-2it}\right) =$$

$$= \frac{1}{3}\left(2e^t + \frac{(-80+40i)}{80}(\cos 2t + i\,\mathrm{sen}\,2t) + \frac{-80-40i}{80}(\cos 2t - i\,\mathrm{sen}\,2t\right.$$

$$= \frac{1}{3}\left(2e^t - 2\cos 2t - \mathrm{sen}\,2t\right)$$

Así.
$$x(t) = \frac{1}{3}\left(e^t + 2\cos 2t + \mathrm{sen}\,2t\right)$$
$$y(t) = \frac{1}{3}\left(e^t - 2\cos 2t - \mathrm{sen}\,2t\right)$$

$t \geq 0$

(por cont.)