MATEMÁTICAS I

Derivadas

Curso 2011/2012

Ejercicio 1 *Calcular las derivadas de las siguientes funciones:*

1. $y = \frac{1}{x^2} \Rightarrow y' = -\frac{2}{x^3}$
2. $y = 2x^3 - 5x^2 - 7x + 4 \Rightarrow y' = 6x^2 - 10x + 7$
3. $y = 5\sin x + 3\cos x \Rightarrow y' = 5\cos x - 3\sin x$
4. $y = \frac{1}{\exp(x)+1} \Rightarrow y' = \frac{-\exp(x)}{(\exp(x)+1)^2}$
5. $y = x^2\exp(x) \Rightarrow y' = 2x\exp(x) + x^2\exp(x) = x\exp(x)(2+x)$
6. $y = \tan^6 x \Rightarrow y' = 6\tan^5 x\frac{1}{\cos^2 x} = 6\tan^5 x\sec^2 x$
7. $y = \tan(\ln x) \Rightarrow y' = \frac{1}{\cos^2(\ln x)}\frac{1}{x} = \sec^2(\ln x)\frac{1}{x}$
8. $y = \ln(\tan\frac{1}{x}) \Rightarrow y' = \frac{1}{\tan\frac{x}{2}}\frac{1}{\cos^2\frac{x}{2}}(\frac{1}{2}) = \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\frac{1}{\cos^2\frac{x}{2}}(\frac{1}{2}) = \frac{1}{\sin\frac{x}{2}}\frac{1}{\cos\frac{x}{2}}(\frac{1}{2}) = \frac{1}{\sin x} = \csc x$
9. $y = \ln(x + \sqrt{x^2+1}) \Rightarrow y' = \frac{1}{(x+\sqrt{x^2+1})}(1 + \frac{2x}{2\sqrt{x^2+1}}) = \frac{1}{\sqrt{x^2+1}}$
10. $y = x^{x^2} \Rightarrow \ln y = \ln x^{x^2} = x^2\ln x \Rightarrow \frac{y'}{y} = 2x\ln x + x^2\frac{1}{x} \Rightarrow y' = y(2x\ln x + x^2\frac{1}{x}) \Rightarrow y = x^{x^2+1}(1 + 2\ln x)$
11. $y = \frac{3}{4}x\sqrt[3]{x} \Rightarrow y = \frac{3}{4}x\sqrt[3]{x} = \frac{3}{4}x^{\frac{4}{3}} \Rightarrow y' = \frac{3}{4}\frac{4}{3}x^{\frac{4}{3}-1} = x^{\frac{1}{3}} = \sqrt[3]{x}$
12. $y = \log_2\sqrt{x} \Rightarrow y' = \frac{1}{2x}\log_2 e = \frac{1}{2x\ln 2}$
13. $y = \log_3(5x^2+1) \Rightarrow y' = \frac{10x}{5x^2+1}\log_3 e$
14. $y = x^{\frac{1}{\ln x}} \Rightarrow \ln y = \ln x^{\frac{1}{\ln x}} = \frac{\ln x}{\ln x} = 1 \Rightarrow \frac{y'}{y} = 0 \Rightarrow y' = 0$
15. $y = \arccos(3x) \Rightarrow y' = -\frac{3}{\sqrt{1-9x^2}}$ *con* $|x| < \frac{1}{3}$

16. $y = \arctan(\ln x) \Rightarrow y' = \frac{\frac{1}{x}}{1+\ln^2 x}$ *con* $x > 0$

17. $y = \sqrt{\tan x} \Rightarrow y' = \frac{1}{2\sqrt{\tan x}} \frac{1}{\cos^2 x} = \frac{\sec^2 x}{2\sqrt{\tan x}}$

18. $y = \arccos(\exp x) \Rightarrow y' = \frac{-\exp x}{\sqrt{1-\exp(2x)}}$ *con* $x \in]-1, 0[$

19. $y = \arctan\sqrt{4x^2-1} \Rightarrow y' = \frac{1}{1+(\sqrt{4x^2-1})^2} \frac{1}{2\sqrt{4x^2-1}} 8x = \frac{4x}{4x^2\sqrt{4x^2-1}} = \frac{1}{x\sqrt{4x^2-1}}$

20. $y = \ln(\tan(\frac{2x+1}{4})) \Rightarrow y' = \frac{1}{\tan(\frac{2x+1}{4})} \frac{1}{\cos^2(\frac{2x+1}{4})} \frac{1}{2} = \frac{1}{2\sin(\frac{2x+1}{4})\cos(\frac{2x+1}{4})} = \frac{1}{\sin(\frac{2x+1}{2})} \Rightarrow$ $y' = \csc(\frac{2x+1}{2})$

Ejercicio 2 *Determinar la derivada de las siguientes funciones:*

1. $y = 3x^3 \ln x - x^3 \Rightarrow y' = 3(3x^2 \ln x + x^3 \frac{1}{x}) - 3x^2 = 9x^2 \ln x$

2. $y = \sqrt{1-3x^2} \Rightarrow y' = \frac{1}{2\sqrt{1-3x^2}}(-6x) = -\frac{3x}{\sqrt{1-3x^2}}$

3. $y = \ln(\frac{x^5}{x^5+2}) \Rightarrow y' = \frac{1}{(\frac{x^5}{x^5+2})}(\frac{5x^4(x^5+2)-x^5 5x^4}{(x^5+2)^2}) = \frac{10}{x(x^5+2)}$

4. $y = x^{\ln x} \Rightarrow \ln y = \ln(x^{\ln x}) = \ln x \ln x \Rightarrow \frac{y'}{y} = (\frac{1}{x}\ln x + \frac{1}{x}\ln x) = 2\frac{\ln x}{x} \Rightarrow$ $y' = 2\frac{\ln x}{x} y = 2\frac{\ln x}{x} x^{\ln x} = 2\ln x x^{-1} x^{\ln x} = 2x^{(\ln x - 1)} \ln x$

5. $y = \log_5(\frac{x+1}{x^2+3}) \Rightarrow y' = \frac{1}{(\frac{x+1}{x^2+3})}(\frac{1(x^2+3)-(x+1)2x}{(x^2+3)^2}) \log_5 e = \frac{3-2x-x^2}{(x+1)(x^2+3)} \log_5 e$ *con* $x > -1$

Ejercicio 3 *Demostrar las siguientes derivadas:*

1. $y = \frac{\sqrt{x}+5}{4x^2+2} \Rightarrow y' = \frac{1-6x^2-40x\sqrt{x}}{\sqrt{x}(4x^2+2)^2}$ *con* $x > 0$

2. $y = \frac{\ln x}{x^5} + \frac{1}{5x^5} \Rightarrow y' = -\frac{5\ln x}{x^6}$

3. $y = \ln(2x^3 + 3x^2) \Rightarrow y' = \frac{6(x+1)}{(2x^2+3x)}$

4. $y = \cos^3 \frac{x}{3} \Rightarrow y' = -\cos^2 \frac{x}{3} \sin \frac{x}{3}$

5. $y = (\frac{2x}{\sqrt[3]{x}-2}) \Rightarrow y' = (\frac{4\sqrt[3]{x}-12}{3(\sqrt[3]{x}-2)^2})$ *con* $x \neq 8$