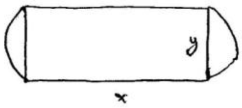
SOLUCION EJERCICIOS DE OPTIMIZACIÓN DEL 7 DE DICIEMBRE DE 2011

1



$A = xy$ función a maximizar

Perímetro $= 2x + 2\left(\pi\left(\frac{y}{2}\right)\right) = 2x + \pi y = 60$
(condición)

$G(x,y,\lambda) = xy + \lambda(2x + \pi y - 60)$ (Ec. de Lagrange)

$$\frac{\partial G}{\partial x} = 0 \rightarrow y + 2\lambda = 0 \rightarrow y = -2\lambda$$

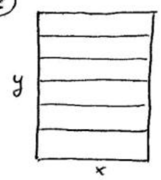
$$\frac{\partial G}{\partial y} = 0 \rightarrow x + \pi\lambda = 0 \rightarrow x = -\pi\lambda \rightarrow 2(-\pi\lambda) + \pi(-2\lambda) = 60$$

$$2x + \pi y - 60 = 0$$

$$\left[\lambda = -\frac{30}{2\pi} = -\frac{15}{\pi}\right]$$

Así $x = -\pi\lambda = 15\,m$
$y = -2\lambda = \frac{30}{\pi}\,m$ $\rightarrow$ $A_{máx} = 15 \cdot \frac{30}{\pi} = 143'24\,m^2$

2



$A = xy$

condición: $30 = 7x + 2y$

$G(x,y,\lambda) = xy + \lambda(7x + 2y - 30)$

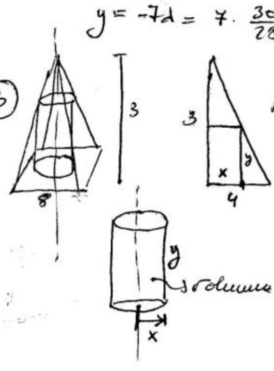
$$\frac{\partial G}{\partial x} = 0 \rightarrow y + 7\lambda = 0 \rightarrow y = -7\lambda$$

$$\frac{\partial G}{\partial y} = 0 \rightarrow x + 2\lambda = 0 \rightarrow x = -2\lambda \rightarrow 7(-2\lambda) + 2(-7\lambda) = 30$$

$$7x + 2y - 30 = 0 \qquad \lambda = -\frac{30}{28}$$

Luego $x = -2\lambda = 2 \cdot \frac{30}{28} = 2'1429\,m$
$y = -7\lambda = 7 \cdot \frac{30}{28} = 7'5\,m$ $\Rightarrow A_{máx} = [xy]_{\text{óptimos}} = 16'07\,m^2$

3



por triángulos semejantes $\Rightarrow \frac{3}{4} = \frac{3-y}{x} \rightarrow \left[y = 3 - \frac{3}{4}x\right]$
condición.

Queremos maximizar Volumen: $V(x,y) = \pi x^2 y$

$G(x,y,\lambda) = \pi x^2 y + \lambda\left(3 - \frac{3}{4}x - y\right)$

volumen $\pi x^2 y$

$$\left\{\begin{array}{l} \frac{\partial G}{\partial x} = 0 \rightarrow 2\pi x y - \frac{3}{4}\lambda = 0 \\ \frac{\partial G}{\partial y} = 0 \rightarrow \pi x^2 - \lambda = 0 \\ y = 3 - \frac{3}{4}x \end{array}\right.$$

De $\left.\begin{array}{l} 2\pi x y = \frac{3}{4}\lambda \\ \pi x^2 = \lambda \end{array}\right\}$ dividiéndolas $\Rightarrow 2y = \frac{3}{4}x \left.\right\} \Rightarrow 6 - \frac{6}{4}x = \frac{3}{4}x$

$$\boxed{x = 8/3 \rightarrow y = 1}$$